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Question Bank

Course Name :- B.Sc-I (CBCS) Subject Name :-Differential Calculus

Sem :- I Paper No :- Mathematics P-I

Q. Choose Correct alternative for each of the following.(1 mark each)

1) If $y = \frac{1}{ax+b}$, then y_n =----------

A) $\frac{n!a^n}{(ax+b)^n}$ B) $\frac{(-1)^n n!a^n}{(ax+b)^n}$ C) $\frac{(-1)^n n!a^n}{(ax+b)^{n+1}}$ D) 0

2) If $\bar{z}$ is a complex conjugate of z, where $z = x + iy$, then $z + \bar{z}$ = ---------

A) x B) y C) $2iy$ D) $2x$

3) The simplified form of $\frac{(\cos\theta+isin\theta)^3}{(\cos\theta-isin\theta)^3}$ is ---------

A) 0 B) 1 C) $\cos 6\theta + isin6\theta$ D) $\cos 6\theta - i\sin 6\theta$

4) Leibnitz's theorem is used to find the nth differential coefficient of --------

A) Trigonometric functions only

B) Exponential functions only

C) Sum & difference of two functions

D) Product of two functions

5) For $z = 4 + 3i$, the value of $R(z^3)$ =----------

A) 44 B) -44 C) 33 D) -33

6) $(r+1)^{th}$ term in the expression of $y_n(uv)$ =----------

A) $\binom{n}{r+1}u_{n-r}v_r$ B) $\binom{n+1}{2}u_{n-r}v_r$

C) $\binom{n}{r}u_{n-r}v_r$ D) $\binom{n}{r-1}u_{n-r}v_r$

7) The value of $\left(sin\frac{\pi}{3} + icos\frac{\pi}{3}\right)^3$ =----------

A) -1 B) 1 C) i D) $-i$

8) If $y = (tan^{-1}x)^2$, then $(x^2+1)^2y_2 + 2x(x^2+1)y_1$ =----------

A) 2 B) -2 C) $2y$ D) $-2y$

9) Using De-Moivre's theorem, $\frac{(1+i\sqrt{3})^6}{(1-i\sqrt{3})^6}$ =----------

A) $e^{-i\pi}$ B) e^{4i} C) $e^{i\pi/2}$ D) $e^{3i\pi/2}$

10) If $y = sinxsin3x$, then $y_n =$

A) $\frac{1}{2}\left[\cos\left(2x+\frac{n\pi}{2}\right)-\cos\left(4x+\frac{n\pi}{2}\right)\right]$

B) $\frac{1}{2}\left[\cos\left(4x+\frac{n\pi}{2}\right)-\cos\left(2x+\frac{n\pi}{2}\right)\right]$

C) $\frac{1}{2}\left[\cos\left(4x+\frac{n\pi}{2}\right)+\cos\left(2x+\frac{n\pi}{2}\right)\right]$

D) None of these

11) The roots of the equation $x^3 - 1 = 0$ are -------

A) $cos\frac{(2k+1)\pi}{3} + i\,sin\frac{(2k+1)\pi}{3},\ k = 0,1,2$

B) $cos\frac{(2k-1)\pi}{3} + i\,sin\frac{(2k-1)\pi}{3},\ k = 0,1,2$

C) $cos\frac{2k\pi}{3} + i\,sin\frac{2k\pi}{3},\ k = 0,1,2$

D) None of these

12) If y is a polynomial of degree $n\ in\ x$ and first coefficient is 2, then y_{n-1} =----------

A) $2(n!)$ B) $2(n!)\ x$ C) $2(n-1)!\ x$ D) none of these

13) Which of the following is true?

A) $cosh^2 x + sinh^2 x = 1$ B) $cosh^2 x - sinh^2 x = 1$

C) $sinh^2 x - \cosh^2 x = 1$ D) $sinh^2 x + cosh^2 x = 1$

14) If $y = sin^{-1} x$, then y_n =----------

A) $n^2 y_n$ B) $-n^2 y_n$ C) $-(n^2+1)y_n$ D) None of these

15) The value of i^i is -------

A) $\frac{\pi}{2}$ B) $\frac{-\pi}{2}$ C) $e^{\pi/2}$ D) $e^{-\pi/2}$

16) If $y = (ax+b)^n$, then y_n =---------

A) na^n B) $n!\,a^n$ C) nab^n D)$n!\,b^n$

17) If $tan(x+iy) = p+iq$,then $tan(x-iy)$ =----------

A) $p - iq$ B) $p + iq$ C) $ip - q$ D) $p - q$

18) The n[th] derivative of $y = cos^2 x$ is --------

A) $2^n cos\left(2x + \frac{n\pi}{2}\right)$ B) $2^{n-1} sin\left(2x + \frac{n\pi}{2}\right)$

C)$\frac{1}{2} + 2^{n-1} sin\left(2x + \frac{n\pi}{2}\right)$ D) $2^{n-1} cos\left(2x + \frac{n\pi}{2}\right)$

19) If $x = \tanh^{-1}\left(\frac{1}{2}\right)$, then the value of $sinh\, 2x$ =----------

A) $\frac{3}{4}$ B)$\frac{4}{3}$ C) 2 D) $\frac{1}{2}$

20) If $y^{1/m} - y^{-1/m} = 2x$,then $(x^2 - 1)y_2 + xy_1$ =----------

A) $m^2 y$ B) $-m^2 y$ C) $\pm m^2 y$ D) None of these

21) If $x = cos\,\theta + i\, sin\,\theta$,then $x^n + \frac{1}{x^n}$ =----------

A) $2\, sin\, n\,\theta$ B) $2\, cos\, n\,\theta$ C)$2(cos\, n\,\theta + i\, sin\, n\,\theta)$ D) None of these

22) If $y = e^{3x}$,then y_n =---------

A) $3^n e^x$ B) $3^n e^{3x}$ C) $3^{n-1} e^x$ D) $3^{n-1} e^{3x}$

23) Which of the following is false?

A)$sinh\, z = \frac{e^z - e^{-z}}{2}$ B) $cosh\, z = \frac{e^z + e^{-z}}{2}$

C) $sinh^2 z - cosh^2 z = 1$ D) $cosh\, z = cos(iz)$

24) The value of $|x - iy|$is -------

A) Non-negative real number B)Non-negative complex number.

C) Negative complex number D) Negative real number

25) For the complex numbers$z_1$and z_2, $arg(z_1.z_2)$ =----------

A) $\arg z_1 + \arg z_2$ B) $\arg z_1 - \arg z_2$

C) $\arg z_1 . \arg z_2$ D) $\arg z_1 / \arg z_2$

26) If $tan(x + iy) = p + iq$, then $tan(x - iy)$ =----------

A) $p - iq$ B) $p + iq$ C) $ip - q$ D) $p - q$

27) The n[th] derivative of $y = cos^2 x$ is --------

A) $2^n cos\left(2x + \frac{n\pi}{2}\right)$ B) $2^{n-1} sin\left(2x + \frac{n\pi}{2}\right)$

C)$\frac{1}{2} + 2^{n-1} sin\left(2x + \frac{n\pi}{2}\right)$ D) $2^{n-1} cos\left(2x + \frac{n\pi}{2}\right)$

28) If $x = \tanh^{-1}\left(\frac{1}{2}\right)$, then the value of $sinh\, 2x$ =----------

A) $\frac{3}{4}$ B) $\frac{4}{3}$ C) 2 D) $\frac{1}{2}$

29) If $y^{1/m} - y^{-1/m} = 2x$, then $(x^2 - 1)y_2 + xy_1$ =----------

A) $m^2 y$ B) $-m^2 y$ C) $\pm m^2 y$ D)None of these

30) If $x = cos\,\theta + i\, sin\,\theta$, then $x^n + \frac{1}{x^n}$ =----------

A) $2\, sin\, n\,\theta$ B) $2\, cos\, n\,\theta$ C) $2(cos\, n\,\theta + i\, sin\, n\,\theta)$ D) None of these

31) If $y = e^{3x}$, then y_n =---------

A) $3^n e^x$ B) $3^n e^{3x}$ C) $3^{n-1} e^x$ D) $3^{n-1} e^{3x}$

32) Which of the following is false?

A)$sinh\, z = \frac{e^z - e^{-z}}{2}$ B) $cosh\, z = \frac{e^z + e^{-z}}{2}$

C) $sinh^2 z - cosh^2 z = 1$ D) $cosh\, z = cos(iz)$

33) For $z = 4 + 3i$, the value of $R(z^3)$ =----------

B) 44 B) -44 C) 33 D) -33

34) If $y = \frac{1}{ax+b}$, then y_n =----------

B) $\frac{n!\ a^n}{(ax+b)^n}$ B) $\frac{(-1)^n\, n!\, a^n}{(ax+b)^n}$ C) $\frac{(-1)^n\ n!\ a^n}{(ax+b)^{n+1}}$ D) 0

35) If $\bar{z}$ is a complex conjugate of z, where $z = x + iy$, then $z + \bar{z}$ = ---------

B) x B) y C) $2iy$ D) $2x$

36) The simplified form of $\frac{(\cos\theta + \ sin\,\theta)^3}{(\cos\theta - \ sin\,\theta)^3}$ is ---------

B) 0 B) 1 C) $\cos 6\theta + isin\, 6\theta$ D) $\cos 6\theta - i \sin 6\theta$

37) Leibnitz's theorem is used to find the n[th] differential coefficient of --------

E) Trigonometric functions only

F) Exponential functions only

G) Sum & difference of two functions

H) Product of two functions

38) The value of $|x - iy|$is -------

B) Non-negative real number

C) Non-negative complex number.

D) Negative complex number

E) Negative real number

39) $(r+1)^{th}$ term in the expression of $y_n(uv)$ =----------

B) $\binom{n}{r+1} u_{n-r}\, v_r$ B) $\binom{n+1}{2} u_{n-r} v_r$

D) $\binom{n}{r} u_{n-r} v_r$ D) $\binom{n}{r-1} u_{n-r} v_r$

40) The roots of the equation $x^3 - 1 = 0$ are -------

A) $cos\frac{(2k+1)\pi}{3} + i\, sin\frac{(2k+1)\pi}{3},\ k = 0, 1, 2$

B) $cos\frac{(2k-1)\pi}{3} + i\, sin\frac{(2k-1)\pi}{3},\ k = 0, 1, 2$

C) $cos\frac{2k\pi}{3} + i\, sin\frac{2k\pi}{3},\ k = 0, 1, 2$

D) None of these

41) If y is a polynomial of degree $n\ in\ x$and first coefficient is 2, then y_{n-1} =----------

B) $2(n!)$ B) $2(n!)\ x$ C) $2(n-1)!\ x$ D) none of these

42) Which of the following is true?

A) $cosh^2 x + sinh^2 x = 1$ B) $cosh^2 x - sinh^2 x = 1$

$C)$ $sinh^2 x - \cosh^2 x = 1$ D) $sinh^2 x + cosh^2 x = 1$

43) For the complex numbers $z_1$and z_2, $arg(z_1.z_2)$ =----------

A) $\arg z_1 + \arg z_2$ B) $\arg z_1 - \arg z_2$

C) $\arg z_1 . \arg z_2$ D) $\arg z_1 / \arg z_2$

44) The value of $i^i$is -------

A) $\frac{\pi}{2}$ B) $\frac{-\pi}{2}$ C) $e^{\pi/2}$ D) $e^{-\pi/2}$

45) The value of $\left(sin\frac{\pi}{3}+icos\frac{\pi}{3}\right)^3$ =----------

B) -1 B) 1 C) i D) $-i$

46) If $y=(tan^{-1}x)^2$, then $(x^2+1)^2y_2+2x(x^2+1)y_1$ =----------

B) 2 B) -2 C) $2y$ D) $-2y$

47) Using De-Moivre's theorem, $\frac{(1+i\sqrt{3})^6}{(1-i\sqrt{3})^6}$ =----------

B) $e^{-i\pi}$ B) $e^{4i\pi}$ C) $e^{i\pi/2}$ D) $e^{3i\pi/2}$

48) If $y=sin\,x\,sin\,3x$, then $y_n=$

E) $\frac{1}{2}\left[\cos\left(2x+\frac{n\pi}{2}\right)-\cos\left(4x+\frac{n\pi}{2}\right)\right]$

F) $\frac{1}{2}\left[\cos\left(4x+\frac{n\pi}{2}\right)-\cos\left(2x+\frac{n\pi}{2}\right)\right]$

G) $\frac{1}{2}\left[\cos\left(4x+\frac{n\pi}{2}\right)+\cos\left(2x+\frac{n\pi}{2}\right)\right]$

H) None of these

49) If $y=(sin^{-1}x)^2$, then $(1-x^2)y_2-y_1$ = ----------

A) 2 B) 0 C) 4 D) -2

50) If $y=(ax+b)^n$, then y_n =---------

A) na^n B) $n!\,a^n$ C) nab^n D)$n!\,b^n$

Q. Long answer question (10 marks each)

1) State and prove Leibnitz's theorem and find n[th] derivative of x^3cosx

2) State and prove De-moiver's theorem and find the value of $(sin\frac{\pi}{6}+icos\frac{\pi}{6})^7$

3) If $y=e^{ax}\sin(bx+c)$ then prove that $y_n=r^ne^{ax}\sin(bx+c+n\theta)$ where $r=\sqrt{a^2+b^2}$ and $\theta=\tan^{-1}(\frac{b}{a})$

4) If z is homogeneous function in x and y of degree n then prove that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=nz$

5) If z is homogeneous of two variable x and y of degree n then show that

$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$

6) If $u = f\left(\frac{x^2}{y^2}\right)$ *show that* $x\frac{\partial u}{\partial x} + 2y\frac{\partial u}{\partial y} = 0$

7) *Find the nth derivative of* $x^3 cosx$

8) *If* $u = \log(x^2 + y^2)$ *then show that* $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

9) *Simplify* $\frac{(cos4\theta - isin4\theta)^2 (cos5\theta - isin5\theta)^3}{(cos3\theta + isin3\theta)^7 (\cos 8\theta - isin8\theta)^5}$

10) *Find the nth derivative of* $\tan^{-1} x$

Q. Short answer question (05 marks each)

1) *Find nth derivative of* $e^x logx$

2) *If* $\cos^{-1}(\frac{y}{b}) = \log(\frac{x}{n})^n$ *show that* $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$

3) *If* $y = (\sin^{-1} x)^2$ *show that* $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$

4) *If* $y = \frac{1}{ax+b}$ *then show that* $y_n = \frac{(-1)^n n! a^n}{(ax+b)^{n+1}}$

5) *Find all the* $5th$ *root of unity* .

6) Find all the value of $(-i)^{\frac{1}{6}}$

7) *If* $x^7 = 1$ *solve using* $De - moiver's\ theorem$

8) *Show that* $sin4\theta = 4sin\theta cos^3\theta - 4sin^3\theta cos\theta$

9) *Express* $tan5\theta$ *in terms of* $tan\theta$

10) *Find the nth derivative of* $\sin^{-1} x$

11) *Find the nth derivative of* $\frac{x^3}{(x-1)(x-2)}$

12) Find the value of $(sin\frac{\pi}{6} + icos\frac{\pi}{6})^6$

13) *If* $z = x^2 y^2$ *find* $\frac{\partial z}{\partial x}$ *and* $\frac{\partial z}{\partial y}$

14) *If* $z = x^3 y^3$ *find* $\frac{\partial^2 z}{\partial x \partial y}$ *and* $\frac{\partial^2 z}{\partial y \partial x}$

15) *find* $\frac{\partial z}{\partial x}$ *and* $\frac{\partial z}{\partial y}$ *where* $z = e^x siny$

16) *If* $y = \frac{1}{x^2-2}$ *then find* y_n

17) *Find the nth derivative of* $\frac{x^3}{(x-1)(x-2)}$

18) *If* $y = e^{ax}$ *then show that* $y_n = e^n e^{ax}$

19) *If* $y = \log(ax + b)$ *then* $y_n = \frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$

20) *Find the extreme value of the function* $u = 2x^3 + xy^2 + 5x^2 + y^2$