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Question Bank

Course Name :- B.Sc-I (CBCS) Subject Name :-Calculus

Sem :- I Paper No :- Mathematics P-II

1) The function $f(x) = x(x+3)e^{-x/2}$ satisfy all conditions of Rolle's Theorem in the interval [-3, 0]. Then the value of c is……

A) 0 B) 1 C) 2 D) -2

2) $\lim_{x\to 0} \frac{1-\cos mx}{x^2}$ =……..

A) $\frac{m}{2}$ B) $\frac{m^2}{2}$ C) $\frac{2}{m}$ D) none of these

3) Right hand limit of $f(x) = 4x + 1$ for $0 \le x < 1$

$= 2x^2 - 1$ for $1 \le x \le 2$ as $\boldsymbol{x \to 1}$ is…

A) 5 B) 3 C) 1 D) none of these

4) The value of the mean value theorem, if $f(x) = 2x^2 + 3x + 4$ in [1, 2] is …….

A) 1 B) 2 C) $\frac{3}{2}$ D) $\frac{2}{3}$

5) Which of the following is not an indeterminate form?

A) $\infty + \infty$ B) $\infty - \infty$ C) $\frac{\infty}{\infty}$ D) $0 * \infty$

6) The infinite fourier series expansion Let $1 - x + x^2 - x^3 + \cdots$

A) $\frac{1}{1+x}$ B) $\frac{1}{1-x}$ C) $\frac{1}{1-x^2}$ D) none of these

7) Geometrically the Rolle's Theorem means that the tangent at point $x = c$ to the curve $y = f(x)$ is…….

A) Perpendicular to x-axis C) intersecting to x-axis

B) parallel to x-axis D) perpendicular to y-axis

8) $\lim_{x\to 0} \frac{5^x - 2^x}{\tan x}$ =……..

A) $\frac{2}{5}$ B) $\frac{5}{2}$ C) $\log\left(\frac{2}{5}\right)$ D) $\log\left(\frac{5}{2}\right)$

9) The $(n+1)^{th}$ term in Maclaurin's series is ……

A) $\frac{x^n}{n} f^n(a)$ C) $\frac{x^n}{n!} f^n(0)$

B) $\frac{x^n}{n!} f^n(a)$ D) $f^n(0)$

10) The Rolles Theorem is not applicable for the function $f(x) = |x|$ in [-2,2] is

A) $f(x)$ Is not Continuous at $x = -2$

B) $f(x)$ Is not Continuous at $x = 2$

C) $f(x)$ Is not Continuous at $x = 0$

D) None of these

11) $1 + x + x^2 + x^3 + \cdots =$…….

A) $\frac{1}{1-x}$ B) $\frac{1}{x+1}$ C) $\frac{1}{x-1}$ D) e^x

12) Value of $\lim_{x\to 0}(\sin x)logx$ is……

A) 0 B) ∞ C) 1 D) -1

13) IF $f(x) = e^x, x \in [0,1]$, then the value of c of L.M.V.T is ……

A) $e - 1$ B) $\frac{1}{e-1}$ C)$e(e-1)$ D) $log\ (e-1)$

14)……..can be deduced from Cauchy's M.V.T.

A) Lagrange's M.V.T. C) Leibnitz's Theorem

B) Rolle's Theorem D) Taylor's Theorem

15) $\lim_{x\to a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}} =$……

A) $\sqrt{2}\sin a$ C) $2\sqrt{a}\cos a$

B) $\sqrt{2a}\sin a$ D) $\sqrt{2a}\cos a$

16) If $f(x) = \frac{sin^2\, ax}{x^2}$ for $x \neq 0$ and $\lim_{x\to 0} f(x)$ is ……

A) a^2 B) a C) 0 D) none of these

17) The value of c, in the mean value theorem, if

$f(x) = x(x-1)(x-2)$; $a = 0$, $b = \frac{1}{2}$ is

A) 1 B) $\left(1 + \frac{\sqrt{21}}{6}\right)$ C)$\left(1 - \frac{\sqrt{21}}{6}\right)$ D) none of these

18) A polynomial function in R ……..

A) is never continuous in R

B) May or may not be continuous in R

C) is always continuous in R

D) is continuous in R except at x=0

19) $\lim_{x\to\infty}\left(1 + \frac{1}{x}\right)$ is equal to

A) 1 B) 0 C) e D) ∞

20) If a function f satisfies the conditions of L.M.V.T. for the interval $[a, b]$ and if $f'(c) = 0$ for all $c \in (a, b)$, then the function f is ……

A) Constant function C) decreasing function

B) Increasing function D) homogeneous

21) Left hand limit of $f(x) = 2^{1/x}$ $as\ x \to 0$ is ……..

A) 0 B) ∞ C) $-\infty$ D) none of these

22) $\lim_{x\to 0} sin\left(\frac{1}{x}\right) =$ …….

A) exists B) is equal to zero C) is equal to ∞ D) does not exists

23) The formula of L'Hospital's rule is

A) $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \left[\frac{f(x)}{g(x)}\right]$ C) $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

B) $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{f(a)}{g(a)}$ D) $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$

24) The value of $\lim_{x\to 0}(cotx)^{sinx}$=…..is

A) 1 B) 0 C) e D) e^2

25) The value of a =…. if the $\lim_{x\to 0} \frac{sin2x+asin}{x^3}$ is finite

A) 2 B) -2 C) 1 D) 0

26) Geometrically, the Roll's Theorem means that the tangent at point $x \in c$to the curve $y = f(x)$ is ……..

A) perpendicular to x-axis B) Parallel to x-axis

C) intersecting to x-axis D) perpendicular to y-axis

27) The infinite series $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots =$……..

A) $cox\ x$ B) $sinx$

C) e^x D) a^x

28) Rolle's Theorem is not applicable to $f(x) = x^2$ in$[0, 2]$, because ……..

A)$f(x)$ is not continuous in [0,2] C) $f(0) \neq f(2)$

B) $f(x)$ is not differentiable in [0,2] D) $f(0) = f(2)$

29) If$f(x) = e^x, x \in [0,1]$, then the value of c of L.M.V.T is ……

A) $e-1$ B) $\frac{1}{e-1}$ C)$e(e-1)$ D) $log\ (e-1)$

30) $\lim_{x\to 1} \frac{\sin \pi x}{x-1} =$ ……..

A)1 B)-1 C)π D) $-\pi$

31) The value of $\lim_{x\to 0} \frac{3^x-2^x}{x}$ =……

A)$log\frac{2}{3}$ B) $log\frac{3}{2}$ C) $log3$ D) $log2$

32) The geometrical meaning of L.M.V.T is that the tangent at $c \in (a,b)$ is …..

A) perpendicular to chord AB

B) Parallel to chord AB

C) intersecting to chord AB

D) not related to chord AB

33) If a function $f(x)$ is continuous in the closed interval $[a,b]$& differentiable in the open interval(a,b), then there exists at least one point $c \in (a,b)$ such that ……..

A) $f(c) = \frac{f(b)-f(a)}{b-a}$ B) $f'(c) = \frac{f(b)-f(a)}{b-a}$

C)$f'(c) = \frac{f(a)-f(b)}{b-a}$ D) $f(c) = \frac{f(a)-f(b)}{b-a}$

34) If the function $f(x)\&g(x)$ are continuous in$[a,b]$ differentiable in $(a,b)\&g'(x) \neq 0$ for any $x\epsilon(a,b)$ such that …….

A) $\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$ B) $\frac{f(c)}{g(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$

C) $\frac{g'(c)}{f'(c)} = \frac{g(b)-g(a)}{f(b)-f(a)}$ D) $\frac{g(c)}{f(c)} = \frac{g(b)-g(a)}{f(b)-f(a)}$

35) Which of the following is a Mean Value Theorem …….

A) Leibnitz Theorem B) Euler's Theorem

C) Taylor's Theorem D) Rolle's Theorem

36) $\lim_{x\to 0} \frac{\log(\tan x)}{\log(\sin x)}$ =…….

A)1 B)$\frac{1}{2}$ C)2 D)$\frac{\pi}{2}$

37) $\lim_{x\to 0} x \log x$ =……

A)1 B) 0 C) -1 D) 2

38) $\lim_{x\to 0} \left[\frac{1}{x}\right]^{2\sin x}$ =……

A)1 B) 0 C) $\frac{\pi}{2}$ D)$\frac{3\pi}{2}$

39) $\lim_{x\to 0} \left[\frac{\tan x}{x}\right]^{1/x^2}$ = …….

A)$e^{1/3}$ B) $e^{1/2}$ C)e^3 D)e^2

40) $\lim_{x\to 0}(\cot x)^{\sin x}$ = …….

A)1 B) 2 C) 3 D) e^2

41) $1 + x + x^2 + x^3 + \cdots$ = ……

A)$\frac{1}{1-x}$ B) $\frac{1}{x+1}$ C) $\frac{1}{x-1}$ D) e^x

42) Rolle's theorem is not applicable to the function $f(x) = |x|$ in$[-2,2]$, since ……

A) $f(x)$ is not continuous at $x = -2$

B) $f(x)$ is not continuous at $x = 2$

C) $f(x)$is not continuous at $x = 0$

D) $f(x)$is not differentiable at $x = 0$

43) If a function f satisfies the condition of L.M.V.T for the interval $[a, b]$ and if $f'(c) = 0$ for all $c\epsilon(a, b)$, then the function f is …….

A) constant function B) Increasing function

C) decreasing Function D) homogeneous

44) Right hand limit of $\lim_{x\to 0} \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x+1}}}$ is …….

A)0 B) 1 C) ∞ D) -∞

45) left hand limit of $\lim_{x\to 0} \frac{1}{x}$ is ……..

A) ∞ B) -∞ C) 1 D) none of these

46) $\lim_{x\to 0} \frac{\cos x - \cos a}{x-a}$ =……

A) $-\sin a$ B)$\sin a$ C)$\cos a$ D)$\sin a \cos a$

47) The infinite series $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots=$.....

A)$\cos x$ B)$\sin x$ C) $-\sin x$ D)$-\cos x$

48) The infinite series $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots=$ …….

A) $log(1+x)$ B)$log\ (1-x)$ C)$\log x$ D)e^x

49) $\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{x-\frac{\pi}{4}}=$ …….

A)$\sqrt{2}$ B) $-\sqrt{2}$ C)$\frac{\pi}{4}$ D) $-\frac{\pi}{4}$

50) $\lim_{x\to 0}\frac{log10+log(x+0.1)}{x}=$ …….

A)10 B)$\frac{1}{10}$ C)1 D)$\frac{1}{100}$

Q.Long answer question (10 marks each)

1) State Rolles theorem and discuss the geometrical meaning

2) State and prove Cauchy's mean value theorem and verify Cauchy mean value theorem for $2x^3$ and x^6 in [a,b] and find C

3) State and prove L'Hospital Rule and solve $\lim_{n\to 0}\frac{e^{ax}-e^{-a}}{\log(1+bx)}$

4) Verify Lagrange's M.V. theorem for
$f(x)=2x^2-7x-10\ Over\ [2,5] and\ find\ C$

5) Verify roll's theorem

i) $f(x)=\frac{x^2-4x}{x+2}$ in [0,4]

ii $f(x)=x(x+3)e^{\frac{-x}{2}}$ in [-3,0]

6) Show that $\lim_{n\to 4}(x^2+7x)=44$

7) Show that $\lim_{n\to 6}\sqrt{x+9=15}$

8) Evaluate $\lim_{n\to 0}\frac{xe^x-\log(1+x)}{x^2}$

9) If a function f is continuous is [a, b] then it is bounded in [a, b].

10) If a function f is continuous in a closed interval [a, b] then it attains its bounds at least onces in [a, b].

Q. Short answer question (05 marks each)

1) Show that the function $f(x) = x^2 + x + 1$ *is continous at* $x = 1$

2) Show that the function $f(x) = cosx$ *is continous for any given value of* x

3) Examine the continuity the following function

$$f(x) = xsin\frac{1}{x} \quad , f(0) = 0$$

4) Evaluate $\lim_{x\to\frac{\pi}{2}}(cosx)^{cosx}$

5) Evaluate $\lim_{x\to 0}(sinx)^{tanx}$

6) Verify Cauchy's Mean Value theorem for the function e^x *and* e^{-x} in [a,b]

7) Discuss the applicability of Cauchy's Mean Value theorem for the function $f(x) =$ x^2 *and* $g(x) = x^3$ in [-2, 3]

8) Show that $\sin hx = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \ldots \ldots \ldots \ldots$

9) Evaluate $\lim_{x\to 0}\frac{\log tanx}{logsinx}$

10) Prove that $\log_{x\to 0} xlogx = 0$

11) Evaluate $\log_{x\to 1} sin\, xlogx$

12) Evaluate $\log_{x\to 0}\frac{logx}{cosecx}$

13) Show that $\log_{x\to -2}(x^2 + 3x) = -2$

14) Show that $\log_{x\to 0}\sqrt{4 - x} = 2$

15) Evaluate $\lim_{n\to 0}\frac{e^{2x} - (1+x)^2}{xlog(1+x)}$

16) Evaluate $\lim_{n\to a}\frac{x^a - a^x}{x^x - a^a}$

17) Show that the function $f(x) = x^2$ is derivable at x=2.

18) Show that $f(x) = |x|$ is not derivable at x=0.

19) Show that the function $f(x) = \frac{|x|}{x}$ for x≠ 0 *and* $f(a) = 0$ has jump discontinuity at x=0 at the height of the jump is 2.

20) Evaluate $\lim_{x\to 2}\frac{x^5 - 32}{x^4 - 16}$