Course Name :- B.Sc-I (CBCS)   Subject Name :- Differential Equations

Sem   :- II   Paper No   :- P-III

Q. Choose correct alternative for each of the following. (1 mark each)

1) The differential equation $y^2 = pxy + f(\frac{py}{x})$ reducess to Clairauts form by
the substitution ---------
   - A) $x = u^2$, $y^2 = v$
   - C) $x = u^2$, $y = v^2$
   - B) $x^2 = u$, $y^2 = v$
   - D) $x^2 = u$, $y = v^2$

2) The solution of the equation $\tan(y-px) = P$ is --------------
   - A) $y = cx - \tan^{-1} c$
   - C) $x = y = cx + \tan^{-1} c$
   - B) $y = cx - \tan c$
   - D) $y = cx_+ \tan c$

3) The equation of the form $y = px + f(p)$ is -------------
   - A) Linear
   - C) Bernoullies
   - B) Clairauts
   - D) De-Moivers

4) The solution of $p(p-1) = 0$ is ----------
   - A) $(y-c)(y-x-c) = 0$
   - C) $(y-c)(x-c) = 0$
   - B) $(y-c)(y+x+c) = 0$
   - D) $(y-c)(y-c) = 0$

5) The equation of the form $\frac{dy}{dx} + p(x)y = Q(x)\, y^n$ is called ---------
   - A) Rulers equation
   - C) Bernoullies equation
   - B) Clairauts equation
   - D) Linear equation

6) The I.F. of the equation $x^2y\, dx - (x^3 + y^3)\, dy = 0$ is----------
   - A) $\frac{1}{x^4}$
   - B) $\frac{1}{y^4}$
   - C) $-\frac{1}{y^4}$
   - D) y^4

7) The solution of the differential equation $px = y + k \log p$ is --------
   - A) $y = cx + k\log c$
   - C) $y = cx - k\log c$
   - B) $y = cx + k\log p$
   - D) $y = cx - k\log p$

8) The value of $\frac{1}{D^2 + 3D + 2} e^{-x}$ is ------------
   - A) $\frac{1}{2} e^{-x}$
   - B) $\frac{1}{2} xe^{-x}$
   - C) xe^{-x}
   - D) None of this

9) $\frac{1}{D^2+a^2} \sin ax = $ --------------

A) $-\frac{x}{2a} \cos ax$ B) $\frac{x}{2a} \cos ax$ C) $-\frac{x}{2a} \sin ax$ D) None of this

10) The P.I. of $(D-1)^2 y = x$ is -----------

A) $x-1$ B) $x+2$ C) x^2+2 D) $1-x$

11) By using substitution $z = \log x$, a homogeneous linear D.E.
    can be reduced to ------

A) second order linear equation

B) linear differential equation with constant

C) Exact differential equation

D) Total differential equation

12) The solution of homogeneous linear equation $x^2 \frac{d2\,y}{dx^2} - x \frac{dy}{dx} - 3y = 0$ is

A) $y = c_1 x^3 + c_2 / x$ C) $y = c_1 e^{3x} + c_2 e^{-x}$

B) $y = c_1 x^3 + c_2 x$ D) $y = c_1 x + c_2 / x^3$

13) The equation $(x^2 - ay) dx + (y^2 - ax) dy = 0$ is ---------

A) Homogeneous   C) variable separable

B) Exact   D) Linear

14) The equation $ydx - (x + 2y^3) dy = 0$ becomes exact
    when multiplied by I.F. = ----------

A) y^2 B) $\frac{1}{y^2}$ C) x^2 D) $\frac{1}{x^2}$

15) The solution of the differential equation $y = px + \sin^{-1} p$ is --------

A) $y = cx + \sin^{-1} c$ C) $y = cx - \sin^{-1} c$

B) $y = cx + \sin c$ D) None of this

16)
    If the D.E. M dx + N dy = 0 is of the form y f(x,y) dx + x f(x,y) dy = 0
    then I.F. is------

A) $\frac{1}{Mx+Ny}$ C) $\frac{1}{Mx-Ny}$

B) $\frac{-1}{Mx+Ny}$ D) None of this

17) $(\sin x \cos y + e^{2x})dx + (\cos x \sin y + \tan y)dy = 0$ is-----------

    A) Second order linear equation

    B) Linear differential equation with constant

    C) Exact differential equation

    D) Total differential equation

18) An equation of the form , $\frac{dy}{dx} + Py = Qy^n$,
    where P and Q are functions of x only is called---------

    A) Linear   C) Bernoullies

    B) Clairauts   D) De-Moivers

19) The I.F. of the equation $\cos^2 x \frac{dy}{dx} + y = \tan x$ is --------
    A) e^x B) $e^{-\tan x}$ C) $e^{\tan x}$ D) e^{-x}

20) An equation of the form $y = px + f(p)$is called------

    A) Linear   C) Bernoullies

    B) Clairauts   D) De-Moivers

21) The solution of the differential equation $y = p (x- 2)+ a/P$ is ------

    A) $y = p (x- 2)+ a/c$ C) $y = c (x+ 2)+ a/c$

    B) $y = c (x- 2)+ a/c$ D) None of this

22) The solution of $d^3 y/dx^3 + 3 d^2 y / dx^2 + 3dy/dx + y = 0$ is---------

    A) $y = (c_1 x^3 + c_2 / x + c_3 x) e^x$ C) $y = c_1 e^{3x} + c_2 e^{-x} - c_3 x$

    B) $y = c_1 x^3 + c_2 x + c_3$ D) $y = (c_1 + c_2 x + c_3 x^2) e^{-x}$

23)
$$\frac{1}{D-a} X = \text{--------------}$$

    A) $e^{ax} \int X \, e^{ax} \, dx$ C) $-e^{ax} \int X \, e^{ax} \, dx$

    B) $e^{ax} \int (-X) \, e^{-ax} \, dx$ D) $e^{ax} \int X \, e^{-ax} \, dx$

24)
$$\frac{1}{(D-2)(D-3)} e^x = \text{---------}$$

    A) $e^x/2$ B) $-e^x/2$ C) e^x D) None of this

25) $\dfrac{1}{D^2 + a^2}$ sin ax = ----------

    A) $\dfrac{-x}{2a}$ cos ax   C) $\dfrac{x}{2a}$ cos ax

    B) $\dfrac{-x}{2a}$ sin ax   D) $\dfrac{x}{2a}$ sin ax

26) The equation $(x^2 - ay) dx + (y^2 - ax) dy = 0$ is ---------

    A) Homogeneous   C) variable separable

    B) Exact   D) Linear

27) The equation $ydx - (x + 2y^3) dy = 0$ becomes exact

    when multiplied by I.F. =----------

    A) y^2 B) $\dfrac{1}{y^2}$ C) x^2 D) $\dfrac{1}{x^2}$

28) The solution of the differential equation $y = px + \sin^{-1} p$ is --------

    A) $y = cx + \sin^{-1} c$ C) $y = cx - \sin^{-1} c$

    B) $y = cx + \sin c$ D) None of this

29)

    If the D.E. $M dx + N dy = 0$ is of the form $y f(x,y) dx + x f(x,y) dy = 0$
    then I.F. is------

    A) $\dfrac{1}{Mx+N}$ C) $\dfrac{1}{Mx-N}$

    B) $\dfrac{-1}{Mx+Ny}$ D) None of this

30) $(\sin x \cos y + e^{2x})dx + (\cos x \sin y + \tan y)dy = 0$ is -----------

    A) Second order linear equation

    B) Linear differential equation with constant

    C) Exact differential equation

    D) Total differential equation

31) An equation of the form , $\dfrac{dy}{dx} + Py = Qy^n$,
    Where P and Q are functions of x only is called---------

    A) Linear   C) Bernoullies

    B) Clairauts   D) De-Moivers

32) The differential equation $y^2 = pxy + f \left(\dfrac{py}{x} \right)$ reducess to Clairauts form by
    the substitution ---------

            A) $x = u^2$, $y^2 = v$ C) $x = u^2$, $y = v^2$

            B) $x^2 = u$, $y^2 = v$ D) $x^2 = u$, $y = v^2$

33)

The solution of the equation tan (y-px) =P is --------------

A) $y = cx - \tan^{-1} c$ C) $x = y = cx + \tan^{-1} c$

B) $y = cx - \tan c$ D) $y = cx\_ + \tan c$

34) The equation of the form $y = px + f(p)$ is -------------
   A) Linear   C) Bernoullies

   B) Clairauts   D) De-Moivers

35) The solution of $p(p-1)= 0$ is ----------
   A) $(y -c) (y - x- c) =0$ C) $(y -c) (x - c) =0$

   B) $(y -c) (y + x + c) =0$ D) $(y -c) (y - c) =0$

36) The equation of the form $\frac{dy}{dx} + p (x)y = Q(x) y^{n}$ is called -------
   A) Rulers equation   C) Bernoullies equation

   B) Clairauts equation   D) Linear equation

37) The I.F. of the equation $x^2y \, dx - (x^3 + y^3) dy = 0$ is----------

A) $\frac{1}{x^4}$ B) $\frac{1}{y^4}$ C) $-\frac{1}{y^4}$ D) y^4

38) The solution of the differential equation $px = y + k \log p$ is --------

A) $y = cx + k\log c$ C) $y = cx - k\log c$

B) $y = cx + k\log p$ D) $y = cx - k\log p$

39) The value of $\frac{1}{D^2 + 3D + 2} e^{-x}$ is ------------

A) $\frac{1}{2} e^{-x}$ B) $\frac{1}{2} xe^{-x}$ C) xe^{-x} D) None of this

40) $\frac{1}{D^2 + a^2} \sin ax =$ --------------
   A) $-\frac{x}{2a} \cos ax$ B) $\frac{x}{2a} \cos ax$ C) $-\frac{x}{2a} \sin ax$ D) None of this

41) The P.I. of $(D - 1)^2 y = x$ is -----------

A) $x - 1$ B) $x + 2$ C) $x^2 + 2$ D) $1 - x$

42) By using substitution $z = \log x$, a homogeneous linear D.E.
   can be reduced to ------
   A) second order linear equation

   B) linear differential equation with constant

   C) Exact differential equation

   D) Total differential equation

43) The solution of homogeneous linear equation $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 3y = 0$ is

A) $y = c_1 x^3 + c_2 / x$

C) $y = c_1 e^{3x} + c_2 e^{-x}$

B) $y = c_1 x^3 + c_2 x$

D) $y = c_1 x + c_2 / x^3$

44) The I.F. of the equation $\cos^2 x \frac{dy}{dx} + y = \tan x$ is --------

A) e^x B) $e^{-\tan x}$ C) $e^{\tan x}$ D) e^{-x}

45) An equation of the form $y = px + f(p)$ is called------

A) Linear   C) Bernoullies

B) Clairauts   D) De-Moivers

46) The solution of the differential equation $y = p(x-2) + a/P$ is ------

A) $y = p(x-2) + a/c$ C) $y = c(x+2) + a/c$

B) $y = c(x-2) + a/c$ D) None of this

47) The solution of $d^3 y/dx^3 + 3 d^2 y / dx^2 + 3dy/dx + y = 0$ is---------

A) $y = (c_1 x^3 + c_2 / x + c_3 x) e^x$ C) $y = c_1 e^{3x} + c_2 e^{-x} - c_3 x$

B) $y = c_1 x^3 + c_2 x + c_3$ D) $y = (c_1 + c_2 x + c_3 x^2) e^{-x}$

48)
$$\frac{1}{D-a} X = \text{-------------}$$

A) $e^{ax} \int X e^{ax} dx$ C) $-e^{ax} \int X e^{ax} dx$

B) $e^{ax} \int (-X) e^{-ax} dx$ D) $e^{ax} \int X e^{-ax} dx$

49)
$$\frac{1}{(D-2)(D-3)} e^x = \text{---------}$$

A) $e^x/2$ B) $-e^x/2$ C) e^x D) None of this

50) $\frac{1}{D^2 + a^2} \sin ax = \text{----------}$

A) $\frac{-x}{2a} \cos ax$ C) $\frac{x}{2a} \cos ax$

B) $\frac{-x}{2a} \sin ax$ D) $\frac{x}{2a} \sin ax$

Q.Long answer question (10 marks each)

1) Define clairaut's equation and explain the method of solving it and solve

$$y = px + \sqrt{1 + p^2}.$$

2) Prove that the necessary and sufficient condition that $Mdx + Ndy = 0$ to   be exact is

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ and solve $(sinxcosy + e^{2x})dx + (cosxsiny + tany)dy = 0$

3) Define Bernoulli's differential equation and explain how they can be solved.

Also solve$(1 - x^2)\frac{dy}{dx} + xy = xy^2.$

4) Differential operator $D = \frac{d}{dx}$ and show that $(D - \alpha)(D - \beta)y = (D - \beta)(D - \alpha)y$

where α and β are constant.

5) Explain the method of solving the linear differential equation $\frac{dy}{dx} + Py = Q$ where P an Q

are function of x and y. Also Solve $\frac{dy}{dx} + \frac{4x}{1+x^2}y = \frac{1}{(1+x^2)^3}.$

6) Show that $\frac{1}{D-a}X = e^{ax} \int xe^{-a} \, dx$ and solve $\frac{1}{D+2}sinx$

7) Explain the method of solving the particular integral of $f(D)y = X$ and Also solve

$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$

8) Show that $\frac{1}{f(D^2)}sinax = \frac{1}{f(-a^2)}sinax$ and also solve $\frac{d^3y}{dx^3} - y = sin2x$

9) Show that $\frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax}$ when $f(a) \neq 0$ and solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$

10) Solve the equation $\frac{d^4y}{dx^4} + 2\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$

Q. Short answer question (05 marks each)

1) Solve $(1 + x^2)\frac{dy}{dx} + 2xy = cosx$

2) Solve $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$

3) Solve $\frac{dy}{dx} - \frac{tany}{1+x} = (1 + x)e^x secy$

4) Solve $p^2 - 5p + 6 = 0$

5) Solve $p^2 + 8p + 15 = 0$

6) Solve $y = 2px + x^2p^4$

7) Solve $p = \tan\left(x - \frac{p}{1+p^2}\right)$

8) Solve $p = \log(px - y)$

9) Solve $(px - y)(x - py) = p$ by using substitution $x^2 = u$, $y^2 = v$.

10) Show that $\frac{1}{D+a}X = e^{-ax}\int xe^{ax}dx$

11) Solve $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$

12) Solve $(x^4 - 2xy^2 + y^4)dx + (4x^4 - 2x^2y - siny)dy = 0$

13) Solve $\left[y\left(1 + \frac{1}{x}\right) + cosy\right]dx + [x(1 - siny + logx)]dy = 0$

14) Solve $x\frac{dy}{dx} - y = x(x^2 - y^2)$

15) Solve $(x^2 + y^2 + 1)dx - 2xydy = 0$

16) Solve $(x^2 - ay)dx + (y^2 - ax)dy = 0$

17) Solve $p^2 - 7p + 12 = 0$

18) Solve $y = -px + x^4p^2$

19) Solve $x = \frac{y}{3p} - 2py^2$

20) Solve $p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0$